A HYBRIDIZABLE INTERIOR PENALTY DISCONTINUOUS GALERKIN METHOD FOR LOCALLY DEGENERATE ELLIPTIC PROBLEMS

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We design a hybrid variant of the famous Interior Penalty Discontinuous Galerkin (IPDG) method to solve degenerate second-order elliptic problems. This hybrid variant denoted H-IP method is exposed in a unified formalism as it can handle (second-order) diffusion problems, (first-order) advection-reaction problems as well as mixed issues combining both previous mechanisms for a wide range of Péclet numbers, including the delicate situation where the diffusion vanishes on a subdomain. Notably, this unusual circumstance can be easily encountered in the context of mass transport in fractured porous media. It is well-known that the presence of fractures deeply affects the process since they represent preferential fluid flow paths. As a result, the global phenomenon may be governed by a hyperbolic partial differential equation in the fracture, and by an elliptic one in the porous matrix. Let us emphasize that such problems are particularly delicate to solve numerically since the scalar variable can jump at the interface separating hyperbolic and elliptic regions [1]. The class of degenerate elliptic problems has been analyzed by different authors during the last decades, mainly in the context of DG methods (see [2] and the abundant references therein). However, the literature is relatively scarce concerning its resolution by Hybridizable Discontinuous Galerkin (HDG) methods, and this is the purpose of the present work. Several assets render the HDG methods attractive: they are amenable to the static condensation, and coupled degrees of freedom are localized at the mesh skeleton leading to a compact stencil. The robustness and accuracy of the proposed H-IP method are investigated through extensive numerical experiments in two-dimensional space for arbitrary polynomial orders k, including the lowest-order case $k = 0$. Optimal convergence of the scalar variable is generally observed for all investigated regimes.

References

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